# Mathemathical Considerations of Early Retirement 

tags: FIRE

## - Deriving the Period until Financial Independence

- Using simple financial mathematics around the future value of an annuity due, we can compute the savings duration until you're financially independent and could, potentially, retire and live happily ever after.
- The future value at retirement here consists of the future value of your net worth today plus the future value of an annuity of your savings rate.
- For simplicity sake we normalize income to 1, of which you save a fraction $s$, each year, for $N$ years. This, as well as your initial net worth $Y_{0}$, is compounded at an interest rate $i$.
- To retire, then, you will need a terminal net worth of $Y_{t} *(1-s)$ : Your net worth in year $t$ , multiplied by your spending rate (the opposite of your savings rate)

$$
Y_{t}(1-s)=Y_{0}(1+i)^{N}+s \frac{(1+i)^{N}-1}{i}
$$

- We multiply by $i$ and rearrange:
- $Y_{t} i(1-s)=Y_{0}(1+i)^{N} i+s\left[(1+i)^{N}-1\right]$
- $Y_{t} i(1-s)=(1+i)^{N}\left[Y_{0} i+s\right]-s$
- Adding $s$ and dividing by $Y_{0} i+s$ gives
- $Y_{t} i(1-s)+s=(1+i)^{N}\left[Y_{0} i+s\right]$
- divide this by $\left(Y_{0} i+s\right)$
- $\frac{Y_{t} i(1-s)+s}{Y_{0} i+s}=(1+i)^{N}$
- $(1+i)^{N}=\frac{Y_{t} i(1-s)+s}{Y_{0} i+s}$
- Afterwards, take the natural log:
$N \ln (1+i)=\ln \left(\frac{Y_{t} i(1-s)+s}{Y_{0} i+s}\right)$
- Then divide by $\ln (1+i)$ :
- $N=\frac{\ln \left(\frac{Y_{t} i(1-s)+s}{Y_{0} i+s}\right)}{\ln (1+i)}$

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- From this we can recognize two things:

1. the higher your savings rates, the fewer years of income $\left(Y_{t} *(1-s)\right)$ you need to accumulate.
2. the higher the real interest rate, the shorter the time to retirement $N$ will be.

1 Linked Reference

Unlinked References

